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FROM AGENT-BASED MODELS TO NETWORK ANALYSIS (AND RETURN): THE POLICY-MAKING PERSPECTIVE

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From Agent-based models to network analysis (and return): the policy-making perspective

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Abstract

An important perspective use of Agent-based models (ABMs) is that of being employed as tools to support decision systems in policy-making, in the complex systems framework. Such models can be usefully employed at two different levels: to help in deciding (policy-maker level) and to empower the capabilities of people in evaluating the effectiveness of policies (citizen level). Consequently, the class of ABMs for policymaking needs to be both quite simple in its structure and highly sophisticated in its outcomes. The pursuing of simplicity and sophistication can be made more effective by applying network analysis to the emergent results. Actually, in today's world the consequences of choices and decisions and their effects on society, and on its organization, are equally relevant. Considering the agent-based and network techniques together, we have a further important possibility. Since it is easier to have network data (i.e. social network data) than detailed behavioral individual information, we can try to understand the relationships between the dynamic changes of the networks emerging from agent-based models and the behavior of the agents. As we understand these connections, we can apply them to actual networks, to try to understand what the behavioral black boxes of real-world agents contain. We propose a simple basic structure where events, scheduled upon time, call upon agents to behave, to modify their context, and to create new structures of links among them. Events are organized as collections of small acts and steps. The metaphor is that of a recipe, i.e. *a set of directions with a list of ingredients for making or preparing something, especially food* (as defined in the American Heritage dictionary). Technically, recipes are sequences of numerical or alphanumerical codes, reported in vectors, and move from an agent to another determining the events and generating the edges of the emerging networks. A basic code will be shown, useful to manage possible applications in different fields: production, health-care scenarios, paper co-authorship, opinion spreading, etc.

0. Introduction – complexity and policy

In the last two decades, complexity economics has reached a considerable scientific cohesion and it is currently one of the most successful endeavors at the frontier of research. The boundary that needs to be crossed is now that of the policy domain.

It is beyond doubt that the ontology and epistemology of complex systems – heterogeneity, interaction, innovation and adaptation – offers new insights both to scholars and policy makers

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(Fontana 2012), however in spite of a considerable number of case studies there is no sign of an emerging unitary theory¹. On the contrary, on the methods side considerable progress has been made.

Among the tools developed in the complexity field, agent-based modeling (hereafter, ABM) and network analysis (hereafter, NA) seem very important in sustaining the process of bringing complexity to bear on the policy world. The former allows modeling a variety of agents and mechanism of interaction in ways that are precluded from mathematical and econometric models; the latter unveil the role in the structure of interaction to the diffusion of the effects of policy, in their efficiency and stability over time.

Moreover, they allow embedding a huge amount of data in user-friendly models – typically software - that improve the transfer of knowledge and competences from the academic world to the policy environment.

While models using these methods are currently thriving, the attempts at applying them jointly are not very frequent (De Caux et al. 2014, Hamill and Gilbert 2009, Edmonds and Chattoe 2005, Kirman and Vriend 2001, Weisbuch et al. 2000). In the paper, we argue that the combination of the two methods can increase enormously the potential of complexity-based policies and we propose a model that operationalizes the merger of the two from an innovative perspective. We conclude by proposing a project for a novel procedure of analysis that can deduce individual behavior from the structure of emerging network thereby diminishing the computational and informational burden that is required to devise policies in complex environments.

The rest of the paper is organized as follows: section 1. discusses the current state of the literature on the joint use of ABMs and NA and emphasizes its potential benefits; section 2. introduces *recipeWorld*, an agent based model that simulates the emergence of a network out of a decentralized autonomous interaction; section 3. illustrates a reverse engineering technique – from data to model – that we are starting to develop and its importance for policy making; section 4. takes a broader perspective on an ABM/NA policy and discusses how it can overcome some limitations of the current approach. Section 5. concludes with some remarks.

1. Agent-based modeling and Network Analysis: the benefits of cross-fertilization

The very definition of a complex system involves structure and patterns emerging from a decentralized autonomous interaction. The exploration of this micro-macro mapping is well suited to

¹ See Geyer and Rihani (2010) for an interesting collection of such studies.

ABMs, but what if the emerging structure is a network?

To put it differently, social, economic and technological networks in the real world are generated through contacts made by individuals pursuing their own end. This is precisely what happens in ABMs, it follows that we can generate easily and sensibly networks through ABMs.

In addition to the above general consideration, the researches on the topic, emphasize a series of limits of NA that could be overcome thanks to the cross-fertilization with ABMs.

The first issue is that of *dynamics*. Caux et al. (2014, 2) point out that much of network theory focuses on static networks,² whereas it is obvious that interaction is dynamic and evolutionary.

The second issue concerns the *behavior of nodes*. NA has to reconcile two different and sometimes apparently irreconcilable aspects: the need of generating a network through appropriate form/severe rules and the need of embedding in such rules a stylized version of meaningful social and economic behaviors. It seems rules that govern the formation of links that the literature in traditional network theory to date employs are usually very straightforward and often lack empirical foundations (see Roth 2007). It follows that, through these models, we can only generate theoretical networks are essentially abstract in nature.

As far as *methods* are concerned, the traditional mathematical modeling of networks encounters a series of problems. Firstly, the scope for actual interaction is very limited since the behavior of the nodes is synthesized in few formal propositions and this is inherited from cellular automata; secondly, because of this limitation, the obvious way to explore the possible set of nodes configurations is by means of combinatorics. This leads to a serious problem in mastering the model, since it has been shown (Johnson and Gilles 2000) that, for instance, a network with eight nodes can generate up to two hundred fifty million different theoretical networks.

Considering the dimension of real world networks, this seems a serious flaw in the possibility of using such models to guide policy decisions. A further consideration is that the use of combinatorics, while mapping all the possible networks, gives no insight about which is more likely to emerge.

To sum up, the process that guides such research is of the following kind: i) take data from real

² See, for instance, Watts and Strogatz (1998) on the formation of Small world networks. Another limitation of their model is that it cannot grow a network from scratch.

world (e.g. social media); ii) observe regularities (i.e. social networks are often of the small world type); iii) generate theoretical networks with desired properties (e.g. stable and efficient networks); iv) measure the distance between theoretical and actual networks by means of network statistics.

Step iv) is of utter importance. Edmonds and Chattoe (2005), stress the weakness of the causal association between measures and the actual properties of the whole network in the name of algorithmic non-compressibility:

“the most individualistic measures (like density) are most likely not to capture the overall “flavour” of the networks but even for obviously structural measures like centrality and cliques, we are still entitled to ask how well these “subnetwork” measures should be expected to capture properties of the whole network.” (2005, 1)

If this is the case, the measures used to perform step iv) might be inaccurate to give an understanding of the social facts that lie behind the network despite the fact that such an understanding that is the ultimate goal of the entire undertaking.

The issues listed so far often show up jointly. For instance, measures of networks can be unreliable due to the inherent dynamic nature of networks. The usual dynamic version of NA consists in generating a series of frames at fixed intervals resting on the assumption that there is continuity between frames (Barnett 2001). Edmons and Chattoe (2005) stress that this might not be the case since, as interaction takes place, individuals change their attributes and their position in the network (Search for Haldane on financial networks. When networks are generated from data, the problem becomes more stringent: the frames can be produced only according to data availability (yearly, quarterly) and this implies the assumption that change takes place with that specific timing. In general, this is a gross simplification that becomes very dangerous when policy measures are concerned.

Let us see how the introduction of ABM can remedy these defects. Firstly, since ABM are inherently dynamic, the problems with static networks is overcome naturally. Secondly, where the modeling of agents is concerned, ABM permits the desired richness of behaviors and attributes that might bridge the gap between agent-nodes and the real world. As for the problems created by combinatorics, they completely disappear within an ABM where the number of agents is limited only by computational power. The number of possible configuration remains, of course, enormous but the problem can be mitigated by establishing a stronger relationship between purposeful micro behaviors and emerging networks). By virtue of the same argument, we can also solve the problems related to

measurement: ABM involves specifying both a set of individual behaviors and the unfolding of the dynamics of social interactions to include the evolution of networks. This means that we can both measure simulated networks in different ways (just as we can do in real networks but on a much larger scale) but also (as we typically cannot do with real networks) investigate whether the network characteristics we choose to measure correspond effectively to the behavior imposed on the agent/node.

In the paper, we investigate the emergence of networks when the nodes themselves – as individuals in an agent-based simulation – choose to form or maintain links. The literature on the topic is concentrated on conceiving formation/sever rules that can create networks with some desired properties in terms of structure – say, small world or scale free – or in terms of efficiency and stability³. Our contribution, takes a different perspective that can complement and enrich the ongoing research scenario. Our aim is not to grow networks with a priori super-imposed features; rather we start from typical socio-economic interactions (i.e. production, exchange, health care, academic cooperation) and track the emerging regularities. From our angle, the emergent network is not an objective but a consequence of interaction.

The difference is of no small importance. As we will explain in what follows, we aim at observing and mapping the emergent network configurations and at studying them without detailed knowledge of the underlying behavior. In order to exemplify the argument, we propose our benchmark model in which agents build networks through a sequence of action-events-interactions and we provide some examples.

2. A *recipeWorld* for economic policy

recipeWorld is an agent-based model that simulates the emergence of networks out of a decentralized autonomous interaction.⁴ The rationale behind it is to offer a few hints to find a framework and a grammar that are flexible and straightforward enough to encompass the widest possible range of purposeful and socially meaningful individual and organizational behavior. This is meant to meet the obvious requirement of generality but is also thought of as a way of making the simulation setting homogeneous over different types of scenarios (e.g. imagine comparing health and labor market policies in different simulations of the same economic system) thereby making the

³ On the topic see the pioneering papers of Aumann and Myerson (1988), Roth and Sotomayor (1989) and Jackson and Wolinski (1996).

⁴ *recipeWorld* is currently a prototype in NetLogo

A previous implementation of the recipe idea (without the network side) already exists in Java Swarm; it is at <http://web.econ.unito.it/terna/jes/>

A new version is under development in SLAPP (Swarm-Like Agent Protocol in Python); SLAPP is at <http://eco83.econ.unito.it/terna/slapp/>

simulation more transparent for both scholars and policy makers.

In order to accomplish this task, we build a simulation platform – recipeWorld – composed of three foundational elements:

- *recipes*⁵ represent a variable number of steps to be taken in order achieve a given end;
- *orders*, are objects representing the end to be pursued (e.g. produce a good). An order contain technical information (e.g. the production steps) and accounting data;
- *agents*, intended as problem solving cores. Each agent – that can active or inactive - is able to perform one or more of the steps required to complete the recipe.

Recipes are coded as strings of numbers – their components. Each number (or, if you want, each label), is related to an act, a sub-routine, of the modeled action.

For instance:

[3 1 7 6] means:

- execute step 3, then
- execute step 1, then
- ...

Recipes can be of any length and can contain subparts with specific structural characteristics, such as:

[1 4 (3 6 5) 8]

where the instructions in parenthesis have to be run in a parallel way; or

[7 4 {10} 9 2]

⁵ The term recipe is typical of industrial economics. A recipe contains data about the properties of actions (e.g. quantity) and their timing (e.g. parallel or sequential) (Terna 2010, 250); see also <http://web.econ.unito.it/terna/jes/>.

where the part in curly brackets has to be run putting together a batch of different recipes to be executed at the same time (e.g. transportation phase, with a minimum quantity to be transported). For instance, these recipes could represent the steps that are necessary to produce according to the demand (order) expressed by the market. The good moves from one production unit to the other (inactive agents) according to the problem solving skill attributed to each unit. Or else, a person (active agent, in this case the subject launching the “order”) is supposed to suffer of a few healthcare problems represented by recipes as above. Those recipes/events will be activated at different moments of this person’s lifetime. In this case, the steps of the recipe are actions to be executed within the healthcare system (a medical examination, a period in a hospital, having surgery, etc.). It is worth nothing that in both cases, in addition to the economic/social relevance of the emergence detected by the traditional ABM there is a network forming. In the first example the order/product/ is moving from a production unit to another, creating a network among the production units; and, in the second example, the patient acting within the healthcare system creates and then uses links among doctors, hospitals, sanitary tests etc.

Let us briefly illustrate an example of code in order to show how the network emerges. The case is that of the order about goods to be produced, moving from factory to factory.

In Figure 1, we see a set of factories, specialized in executing different steps of each order/recipe. Orders are randomly generated and temporarily stored in an abstract place, as indicated.

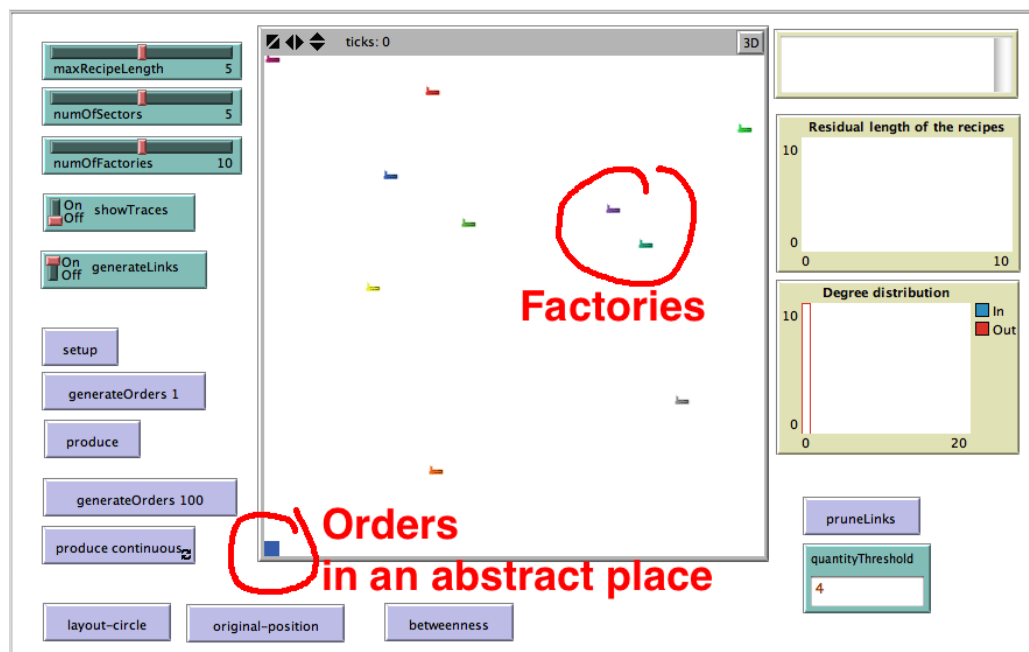


Figure 1

In Figure 2, production takes place and the orders move around the simulated world, generating the links between factories (graphs can be undirected). In order to detect the strength of links, links have an attribute – *quantity* -- measuring how many times the link has been strengthened by orders/recipes passing there.

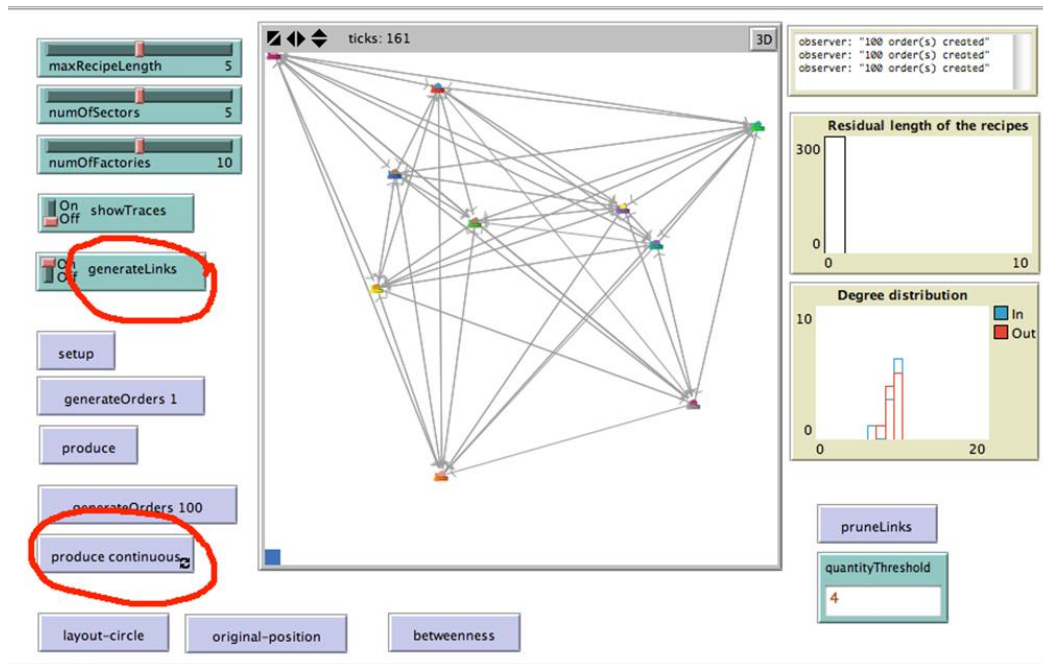


Figure 2

We then simplify the network -- Figure 3-- , by pruning the links having the *quantity* attribute less than or equal to a given value (4 in this case).

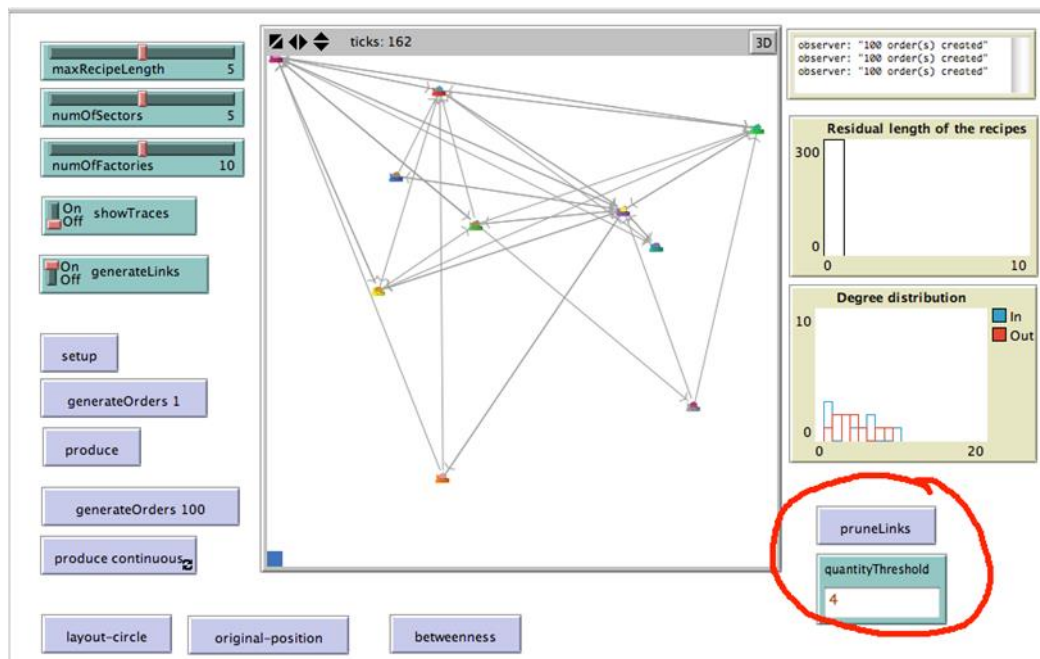


Figure 3

We then report the same network – Figure 4 --, shown in a circle to better identify the system of the links and to calculate the betweenness measure for each node; immediately we can discover the key nodes of the network in a specific run of the simulation.⁶

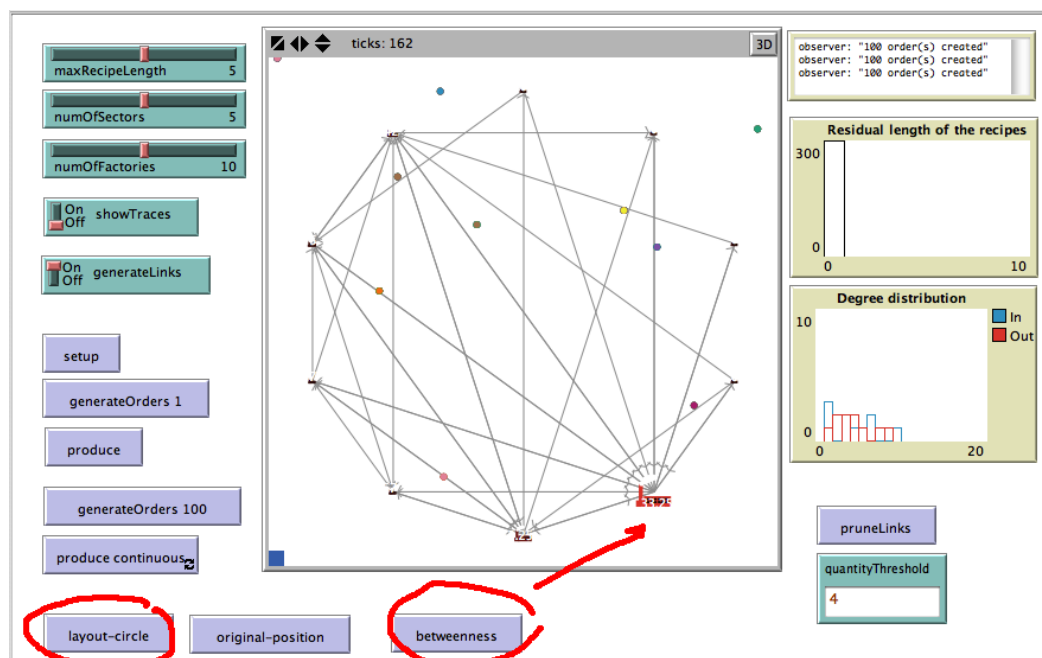


Figure 4

Finally we, restore the original positions of the agents/nodes, showing that it is, in any case,

⁶ Calculations are made using the new NW NetLogo extension
<https://github.com/NetLogo/NW-Extension>

possible to calculate and to display the indicators based on the network algorithms. The original positions are important in spatially critical networks, e.g. a network of hospitals.

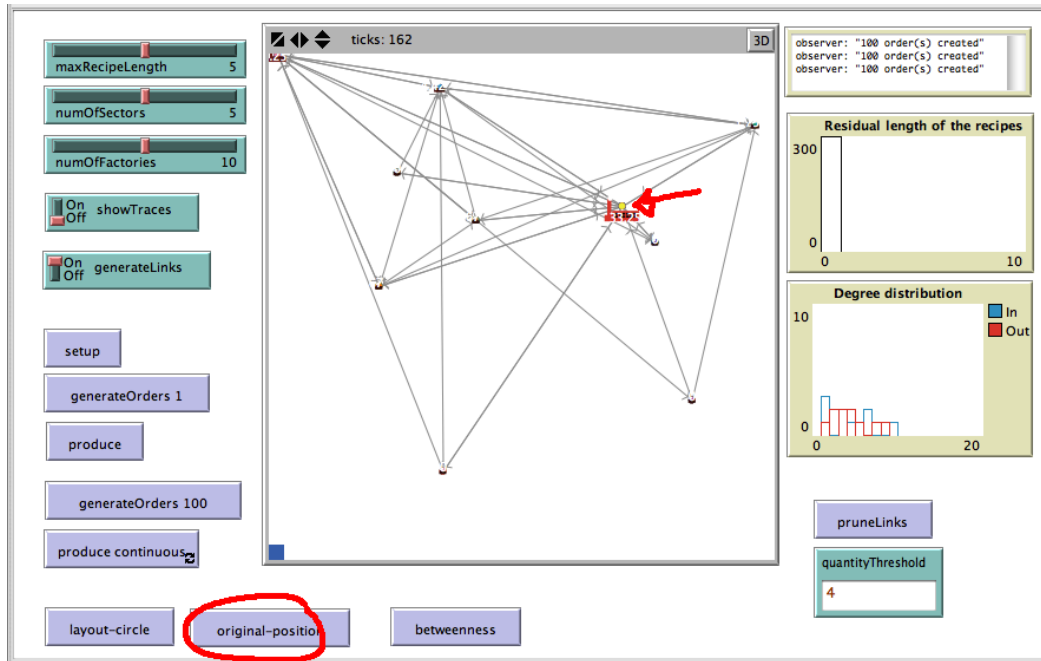


Figure 5

This approach innovates with respect to other interesting works on ABM and networks (e.g. Hamill and Gilbert, 2009; De Caux et al., 2014) where the agents *act to generate* the network; in our context, agents are activated, following their internal rules and capabilities, by the events, and the network emerges as a side effect, as in the real world⁷.

Moreover, with respect to the other approaches (Carayol et al. 2008) that attempt to build networks through various characteristic recombination of links, our approach has no intrinsic limitation in the number of agents to be modeled, in that it does not work in combinatory terms. It follows that it can be used on a scale that can satisfactorily approximate real world phenomena. In addition, the possibility of linking ABM and NA in such a straightforward way is ripe with implications for policy analysis. Let us see them in more detail.

3. A recipeWorld for economic policy: exploring reverse engineering

In figure 6. we represent the relationships between the real world, the simulated world and the conclusions that can be drawn from the model. (A) is the actual world populated by entities e_1, e_2, \dots, e_n and by their actual network; (B) is an ABM where agents a_1, a_2, \dots, a_n are mimicking the actual

⁷ The idea of using the technique of the recipes to wire a network of agents can be found also in Jesi and Fioretti (2012) and it is implemented in the related code on line at <http://aesop-acp.sourceforge.net>

behavior of the entities in (A) via the execution of orders and recipes. (C), represent the agents

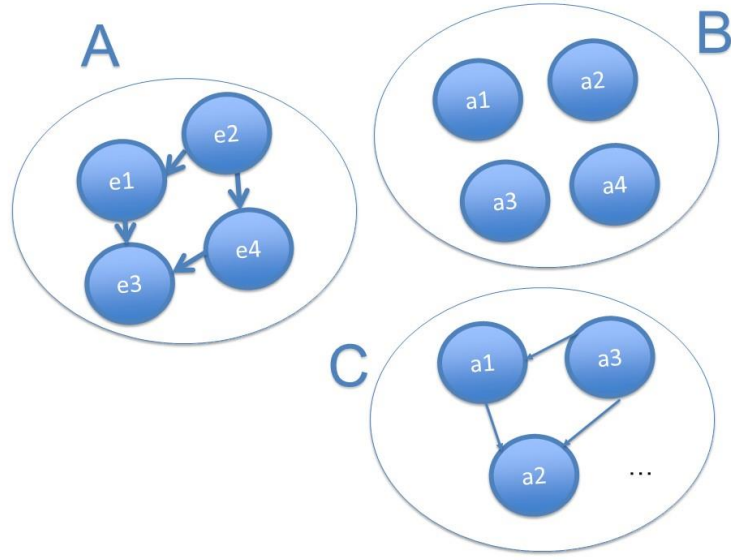


Figure 6

of (B) generating a network, that – if the agents a_i construction is correctly managed – is similar, or very close, to that in (A).

Unfortunately, we do not always have perfect knowledge of the actual world: let us imagine that we do not have thorough knowledge of A, so that the easy construction of the ABM, (B), is not possible. Partial knowledge is a very common situation in research; the problems deriving from it are of different importance and nature according the aim of research. In ABMs, often they manifest themselves in the form of a mapping from many to one, i.e. when various hypothetical micro behaviors generate the same macro regularity, or they might take the form of an excessive simplification of behaviors (KISS principle) in order to keep the parameters' space and the interpretation of causal relationships feasible.

In the case of policy prescriptions, where the necessity of reproducing real world behavior and interaction is particularly felt, the problem is quite acute.

The aim of the future developments of our model is to overcome the impasse – summarized in figure 7 – by means of an innovative statistical procedure.

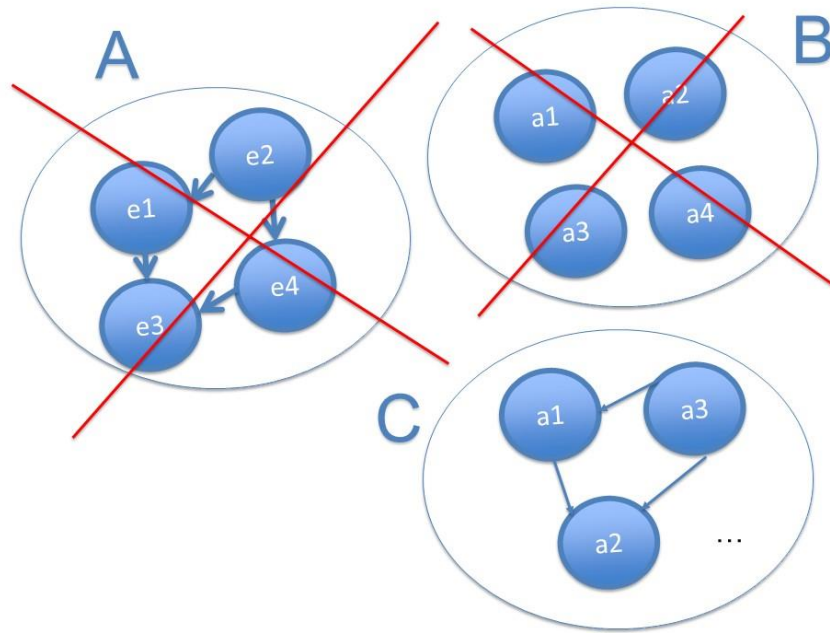


Figure 7

Assume that we know (D) that is data on the network, instead of the exact list of e_i . The intuition behind the project is that by knowing (D) we can infer about C⁸(Figure 8). The idea is to move from (D) to (C), by building an artificial network emerging from a system of agents

⁸ The first work that we have read on the (D) to (C) process is the PhD thesis of Simone Gabbriellini (2009).

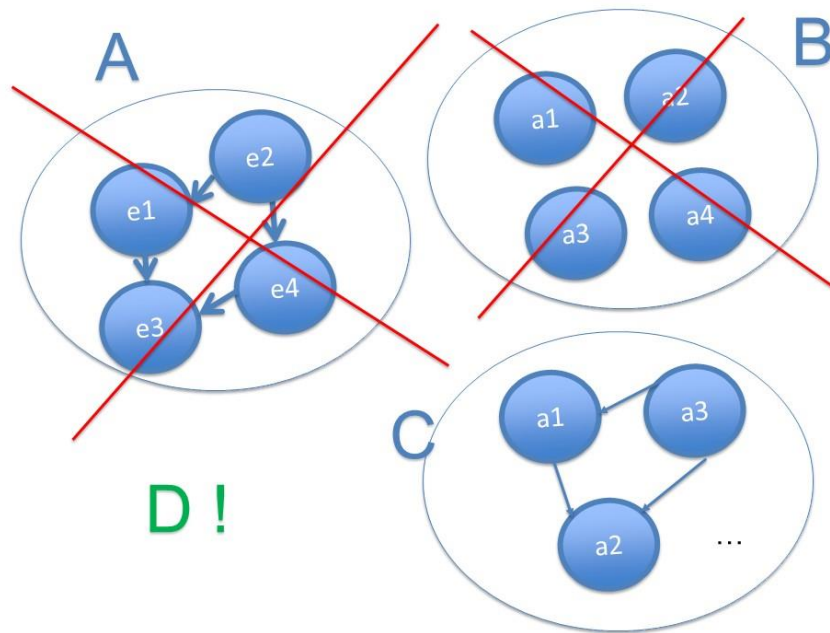


Figure 8

If we succeed in this operation we can be very close to (B) and therefore to (A) (Figure 9).

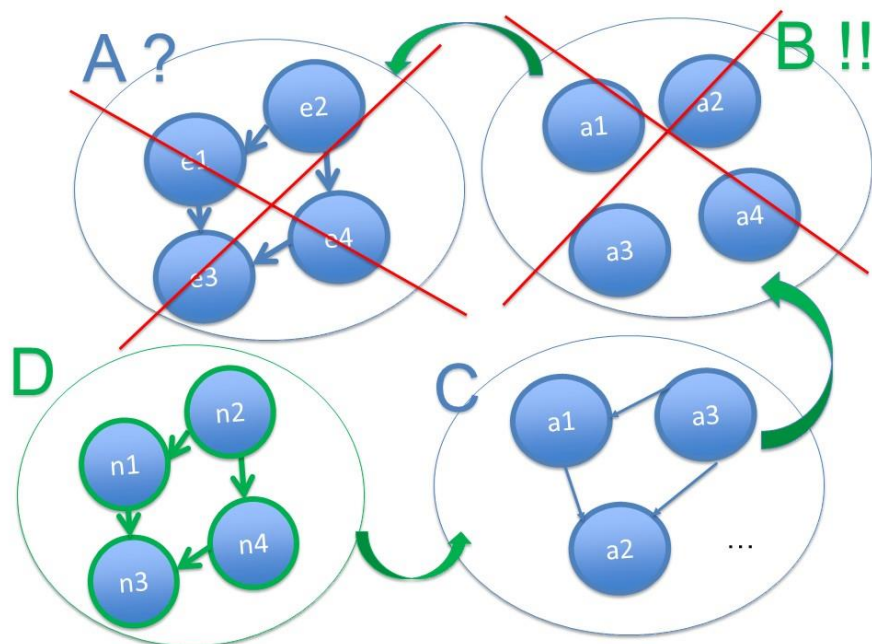


Figure 9

The methodological work to be done is huge, now we move only from (A) to (B) and to (C), using the recipes tool the have (C) emerging from (B). The crucial point in the model is that the network is emerging, not engineered ad hoc

We stress this last sentence: we have interesting works on ABM and networks (Hamill and Gilbert, 2009; De Caux et al., 2014) where the agents *act to generate* the network; in our context, agent are activated, following their internal rules and capabilities, by the events, and the network emerges as a side effect, as in the real world. We can apply to our (C) structures exactly the same range (*palette*) of algorithms that we can apply to (D).

As illustrated by Table 1, that compares traditional NA with ABM-NA, the process that underlies the two kinds of modeling is radically different.

Steps	Network Analysis	Agent-based Network Analysis
I	Take data from real world	Take data from real world (on micro-behavior of network)
II	Observe regularities	Build an ABM of the phenomenon of interest
III	Generate theoretical networks with the desired properties	Observe emerging networks (if any)
IV	Measure the distance between theoretical and actual networks by means of network statistics	Study the dynamic properties of the emerging network.

Table 1

The same marked difference can be found in the output. ABM-NA produces a model that can be used to simulate the long-term effect of policy when individuals and their network adapt to it or introduce new behavior.

4. The broader policy-making perspective

Taking a broad complexity view on policymaking takes us very far from what has been done so far in the field. Even after disenchantment with the beneficial and efficient properties of the self-organization of market, economists have trusted their abilities to control the economy. The idea of intervening in the economy when it fails to adjust spontaneously or when there is the need to steer it toward a (politically) given direction has dominated economics in the last century independently of the prevailing theoretical background. In fact, the various schools of economic thought differ mainly in the prescribed control tools (e.g. monetary vs. fiscal policy), sharing unfaltering confidence in the idea that economies work as machines and that ‘equilibrium’ is the key to their functioning.

By ‘control’ we mean the possibility of adjusting, according to the prescriptions of the various

economic theories, some given variables such as public expenditure or the quantity of money with the aim of obtaining full employment and stable prices. Implicitly, control requires the possibility of forecasting both the trend and the turning points of economic systems. Sadly enough, the history of control and prediction of economic phenomena is beset with failures. The record of failures is as long as the discussion concerning their causes.

Switches in policies are the consequences of this debate: theories used by economists have been held responsible for the ineffectiveness of their applications. A classical example is the discussion generated by the Lucas critique (1976) on large-scale macro-econometric models. He raised a crucial issue: the parameters of those models vary with the undertaken policies (they are structural) and therefore their predictions are likely to be misleading. Lucas' suggestion was to model the micro parameters of the models, that is to say preferences, technology constraints and so forth, in order to understand what the agent would do as a consequence of a policy. The aggregation of individual responses would have generated the macroeconomic impact of the change in policy. Kidland and Prescott (1977) developed Lucas' thesis by operationalizing the search for micro foundation of macroeconomic models.

Dynamic stochastic general equilibrium (DSGE) models constitute the most recent development of this line of research. With respect to previous efforts they try to include historical time and random events. However, in order to assure solvability and simplicity DSGE usually neglects parts of the economic systems such as the financial markets and the banks whose importance has been remarkably highlighted by the last economic crisis.

With the persistence of the current financial crisis and resultant recession that models – especially DSGE – have failed to capture, the discussion concerning the need for new economic theories has gained new vigor. Complexity economics enters the stage by formulating the hypothesis that the cause of the policies failures is not to be found in theories; rather it resides in their underlying ontology. It is the assimilation of the economy to a machine ruled by equilibrium that deceives economists. If we remove this cognitive habit, the importance of complexity-based policy is evident. It allows for procedural-rationality, for explicit institutional settings, for the inclusion of historical time and it permits thorough comparisons among systems. All these features are definitely of immeasurable value to policy makers and their demand for non-conventional tool is now increasing⁹.

⁹ For an interesting survey of policy makers' statements that support this view see Beinhocker (2012).

The joint application of ABM and NA can meet this need by providing a series of information that were hardly available beforehand. By strengthening the connection between micro behavior and emerging networks, agent-based networks can improve knowledge on how efficient and stable¹⁰ networks come about. It is well known that the sets of efficient and stable networks do not always intersect (Carayol et al. 2008, Jackson and Wolinski 1999). The trade-off between the two is of crucial interest to the policy maker when it is a matter of creating a new or modifying an existent network.

In the absence of a way to model the real process of network emergence, scholars have often focused on notions of stability that do not depend on any particular formation process (e.g. pairwise stability), thereby separating the stability of the network from the stability of internal dynamics. We strongly believe that policy could profit from a deeper knowledge of how stability relates to the rules that generate the network.

Notice that agent-based network models can also explore the tension between stability and dynamics. As we explained in section 1, as interaction takes place in the ABM, agents/nodes change their attitudes, features and position in the network. The fusion we have suggested so far let us investigate what happens to the stability of the network when agents, change, eventually disappear and are replaced by new nodes.¹¹ For instance, De Caux et al. (2014), show that for given value of some critical parameter (in their case, movement ability and age of agents) the number of separate clusters in the network decreases sharply and generate mega cluster¹². Such transitions carry important implications for the properties of the networks, such as their resilience to random shocks, which are of crucial importance to policy maker.

As knowledge in this field accumulates, it might be thought that policy maker could fine tune efficiency and stability. In the same sense, but more generally, agent-based network gives insights into the evolution of network statistics over time and on their possible evanescence therefore overcoming some of the problems raised by Edmonds and Chattoe (2005). Among those, thanks to the transparency of the formation process and of the status of the network nodes, there is the possibility of matching the conventional network measures with a more “customized” analysis that can grasp the

¹⁰ The notion of efficiency requires the specification of an external aggregate value such as total productivity, income etc..., while stability is concerned with the allocation of the above value among nodes. When nodes have the ability to form and sever links, stability is calculated with respect to the external value by taking into account the individual incentive to form a link.

¹¹ See for some interesting reflections on this topic Davidsen et al. (2002).

¹² Gonzales et al. (2006) find a critical value for the number of contacts that a node can have in its life and interpret the transition to mega cluster as a percolation process.

actual conditions of (groups) of nodes that are of particular interest (regions, coalitions, productive sectors).

Finally, the technique of reverse engineering that we are starting to tackle in the paper is likewise useful in order to diminish the knowledge that policy makers must acquire in order to act.

5. Concluding remarks

Complexity economics is currently facing the challenge of developing theory and tools that can support decision systems in policy-making. Agent-based modeling plays a crucial in completing this task. ABMs can be useful both in deciding (policy-maker level) and in empowering the capabilities of people in evaluating the effectiveness of policies (citizen level). Consequently, the class of ABMs for policymaking needs to be both quite simple in its structure and highly sophisticated in its outcomes. As we have shown, the application of network analysis to the emergent results can facilitate the achievement of this task by emphasizing the consequences of choices and decisions on the structure of society.

In order to demonstrate the benefits of the matching between ABM and NA we introduce a simple model - *recipeWorld* - in which networks emerge as a result of meaningful economic behavior. We then discuss the implications of the joint use of the two techniques at length, focusing on the role of dynamic network models in policymaking and by introducing a research challenge that we are undertaking. Since it is easier to have network data (i.e. social network data) than detailed behavioral individual information, we can try to understand the relationship between the dynamic changes of the networks emerging from agent-based models and the behavior of the agents. As we understand these relationships, we can apply them to actual networks, trying to understand the content of the behavioral black boxes of real-world agents.

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